Practical development and application of fragility functions

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SPTPS: Nonlinear Structural Analysis, Theory & Applications
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References

Early work on seismic fragility functions

- Kennedy, R.P., C.A. Cornell, R.D. Campbell, S. Kaplan, and H.F. Perla, 1980. Probabilistic seismic safety study of an existing nuclear power plant. *Nuclear Engineering and Design*. 59: 315-338
- Kennedy, R.P., and M.K. Ravindra, 1984. Seismic fragilities for nuclear power plant risk studies. *Nuclear Engineering and Design* vol. 79 (1984). Elsevier Science Publishers B.V., 47-68

ATC-58 and 2nd-generation performance-based earthquake engineering

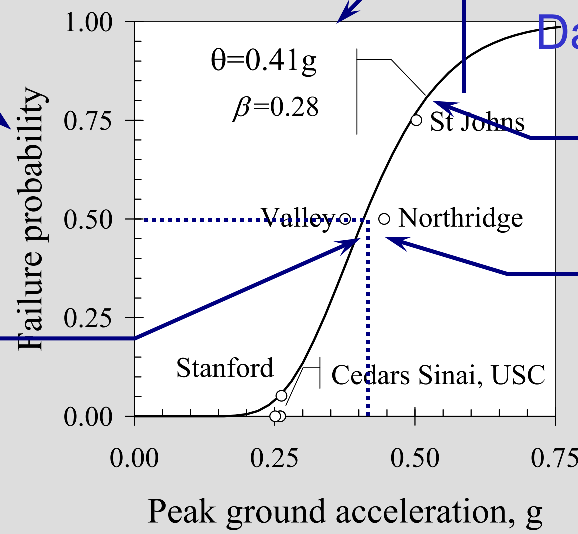
- (ATC) Applied Technology Council, in progress. ATC-58: Guidelines for Seismic Performance Assessment of Buildings, 50% Draft. Redwood City, CA.

Deriving fragility functions for ATC-58

- Porter, K.A., R.P. Kennedy, and R.E Bachman, 2007. Creating fragility functions for performance-based earthquake engineering. *Earthquake Spectra*. 23 (2), May 2007, pp. 471-489, http://www.sparisk.com/pubs/Porter-2007-deriving-fragility.pdf
- Porter, K.A., G. Johnson, R. Sheppard, and R. Bachman, 2010. Fragility of mechanical, electrical, and plumbing equipment. *Earthquake Spectra* 26 (2) 451-472, http://www.sparisk.com/pubs/Porter-2010-MEP-fragility-1.pdf

What's a fragility function?

Probability that a component reaches or exceeds a damage state as a function of demand, e.g., floor acceleration, drift ratio, plastic hinge rotation…



Component: *hydraulic elevator*
Damage state: *nonfunctional*

Probability that elevator rendered nonfunctional

Lognormal CDF fit to data

50% probability of loss of functionality at PGA = 0.41g

Data: Stanford U, various hospitals in Loma Prieta and Northridge earthquakes

Demand: base acceleration (=PGA)

Lognormal fragility function

$$F_{dm}(r) = \Phi\left(\frac{\ln(r/\theta)}{\beta}\right)$$

Φ = Standard normal CDF, e.g., normsdist()

Other distributions okay, LN good because:

1. Fits various structural and nonstructural component failure data well
2. Strong precedent in seismic risk analysis
3. Minimum-information distribution given positive capacity, θ, and β

Challenge

- Creating fragility functions has been an art with no art school
- Comprehensive, standard procedures needed
  - Data come in numerous forms
  - Transparent, consistent, vetted methodologies
- Assign quality levels to fragility functions, e.g.
  - Lots of data, peer-reviewed analysis,
  - Purely analytical, but peer reviewed, or
  - Expert opinion from 1 or 2 experts

Six kinds of data

A. **Actual demand data**: specimens tested with slowly increasing EDP to failure, EDP at failure is known.
B. **Bounding demand data**: specimens observed in lab or field, some failed, some not. Max EDPs are known.
C. **Capable demand data**: specimens tested in lab, none failed, max EDP for each is known.
D. **Derivation**: estimate capacity with structural analysis.
E. **Expert opinion**: capacity from engineering judgment.
U. **Updating**: Bayesian updating of existing fragility function with new type-A, B, or C observations.

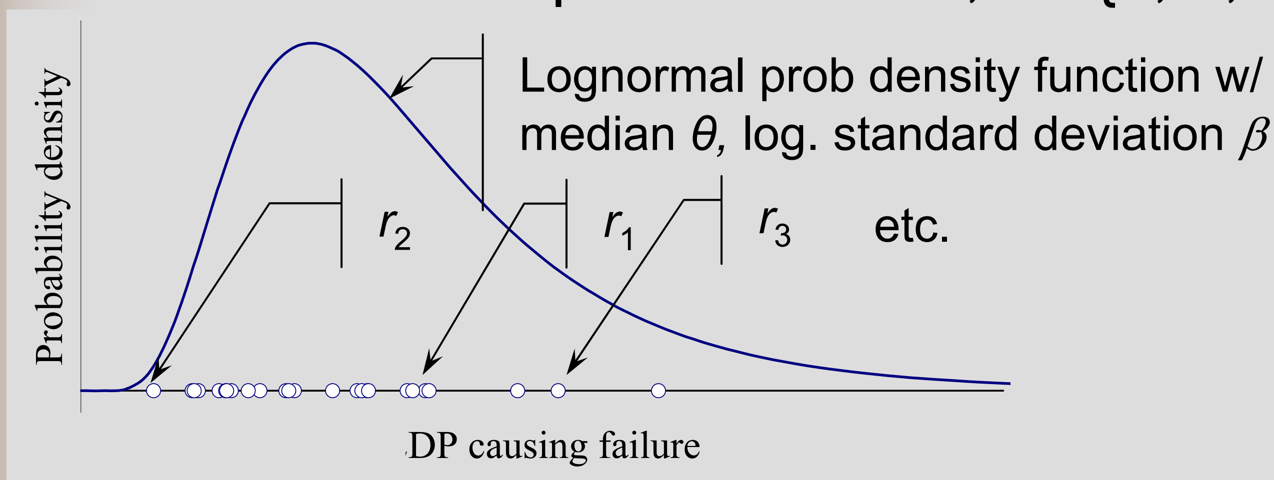
Method A

Known failure DP, each specimen

- r_i = failure DP observed for specimen *i*.

  where *M* = specimens tested to failure

  i = specimen index, $i \in \{1, 2, \ldots M\}$



i	r_i
1	1.42
2	0.39
3	0.59
4	0.32
5	0.53

…

- Calculate parameters of probability distribution:

$$\theta = \exp\left(\frac{1}{M}\sum_{i=1}^{M}\ln r_i\right)$$

$$\beta_d = \sqrt{\frac{1}{M-1}\sum_{i=1}^{M}\left(\ln\left(r_i/\theta\right)\right)^2}$$

Some sampling issues

- Data can be drawn from a sample that differs from the population
  1. Data quality, i.e., quality of the source data, considering number of data points, coverage over the range of damage states, how well constrained are the coordinates of the observations; number of independent observers, and means of observing engineering demands and performance;
  2. Data relevance, i.e., how well the data matches or envelopes the conditions that will be encountered in the population of buildings of the types examined and the diversity of exposed types; the degree to which there may be bias in the selection of observations

Accounting for quality & relevance

$$\beta = \sqrt{\beta_d^2 + \beta_u^2}$$

β_u = 0.25 if data are unrepresentative of actual conditions, e.g., if:

- Test data are available for five (5) or fewer specimens.
- In an actual building, the component can be installed in a number of different configurations, however, all specimens tested had the same configuration.
- All specimens were subjected to the same loading protocol.
- Actual behavior of the component is expected to be dependent on two or more demand parameters, e.g. simultaneous drift in two orthogonal directions, however, specimens were loaded with only one of these parameters.

β_u = 0.1 otherwise

$0.2 \leq \beta \leq 0.6$

Method B

Known max-DP, some failed, some not

COMPILE FAILURE DATA

- r_i = DP experienced by specimen i
- f_i = specimen i failed (1 = yes, 0 = no)

  where M = specimens observed, i = specimen index, $i \in \{1, 2, \dots M\}$

SORT & BIN

- Sort specimens by DP
- Create N bins of $\sim M^{1/2}$ specimens each; j = bin index
- Calculate $\bar{r}_j$, average DP of specimens in bin j
- Calculate ~failure fraction = $(m_j + 1)/(M_j + 1)$, each bin j
  - m_j = number of failed specimens in bin j
  - M_j = total number of specimens in bin j

FIT A LINE AS ON PROBABILITY PAPER

- Transform coordinates to lognormal probability paper:
  - $x_j = \ln(\bar{r}_j)$, each bin j
  - $y_j = \Phi^{-1}((m_j + 1)/(M_j + 1))$
- Fit line to (x_j, y_j) data: $\hat{y} = bx + c$
  - $\beta = 1/b$
  - θ = exp(x-intercept) = $\exp(-c\beta)$

Method B, fitting a line y = bx +c

$$\bar{x} = \frac{1}{M}\sum_{j=1}^{M} x_j$$

$$\bar{y} = \frac{1}{M}\sum_{j=1}^{M} y_j$$

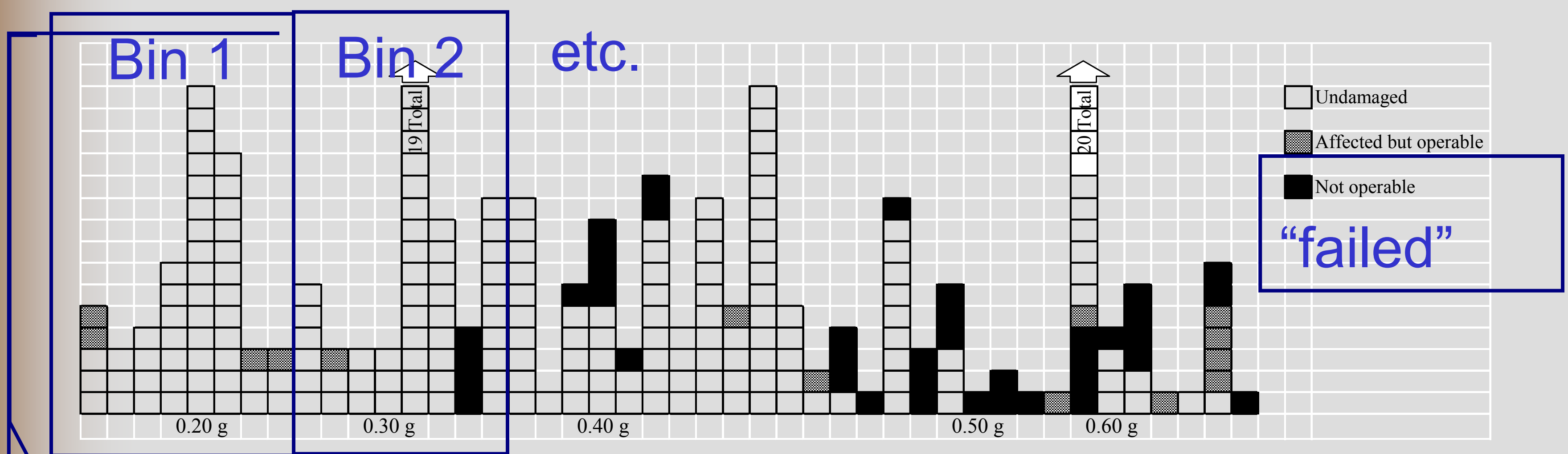
$$b = \frac{\sum_{i=1}^{M}\left(x_j - \bar{x}\right)\left(y_j - \bar{y}\right)}{\sum_{i=1}^{M}\left(x_j - \bar{x}\right)^2}$$

$$\beta_d = \frac{1}{b} = \frac{\sum_{i=1}^{M}\left(x_j - \bar{x}\right)}{\sum_{i=1}^{M}\left(x_j - \bar{x}\right)\left(y_j - \bar{y}\right)}$$

$$\theta = e^{-c\beta_d} = e^{\left(\bar{x} - \bar{y}\beta_d\right)}$$

Method B

Known max-demand, some failed, some not



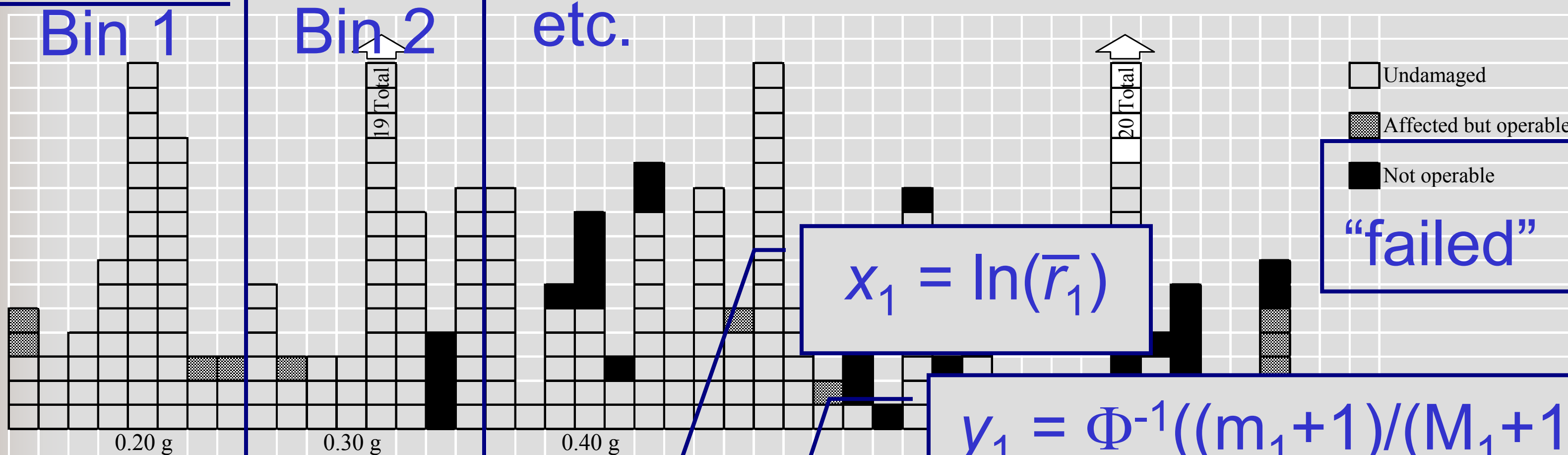
~0.2g: 0/52 failed = 0% failure rate (0 black-filled boxes, 52 boxes)

~0.3g: 4/48 failed = 8.3% failure rate

~0.4g: 8/84 failed = 9.5%

~0.5g: 15/35 failed = 43%

Method B illustration



$x_1 = \ln(\bar{r}_1)$

$y_1 = \Phi^{-1}((m_1+1)/(M_1+1))$

j	a_j (g)	$\bar{r}_j$ (g)	M_j	m_j	x_j	y_j	$x_j - \bar{x}$	$y_j - \bar{y}$	$(x_j - \bar{x})^2$	$(x_j - \bar{x})(y_j - \bar{y})$
1	0.15	0.2	52	0	-1.61	-2.08	-0.623	-1.031	0.388	0.642
2	0.25	0.3	48	4	-1.20	-1.27	-0.217	-0.223	0.047	0.049
3	0.35	0.4	84	8	-0.92	-1.25	0.07[illegible]	[illegible]202	0.005	-0.014
4	0.45	0.5	35	15	-0.69	-0.14	0.29[illegible]	[illegible]907	0.086	0.266
5	0.55	0.6	41	12	-0.51	-0.50	0.47[illegible]	[illegible]549	0.226	0.261
Σ =			260		-4.93	-5.23			0.753	1.204
Avg =					-0.99	-1.05				

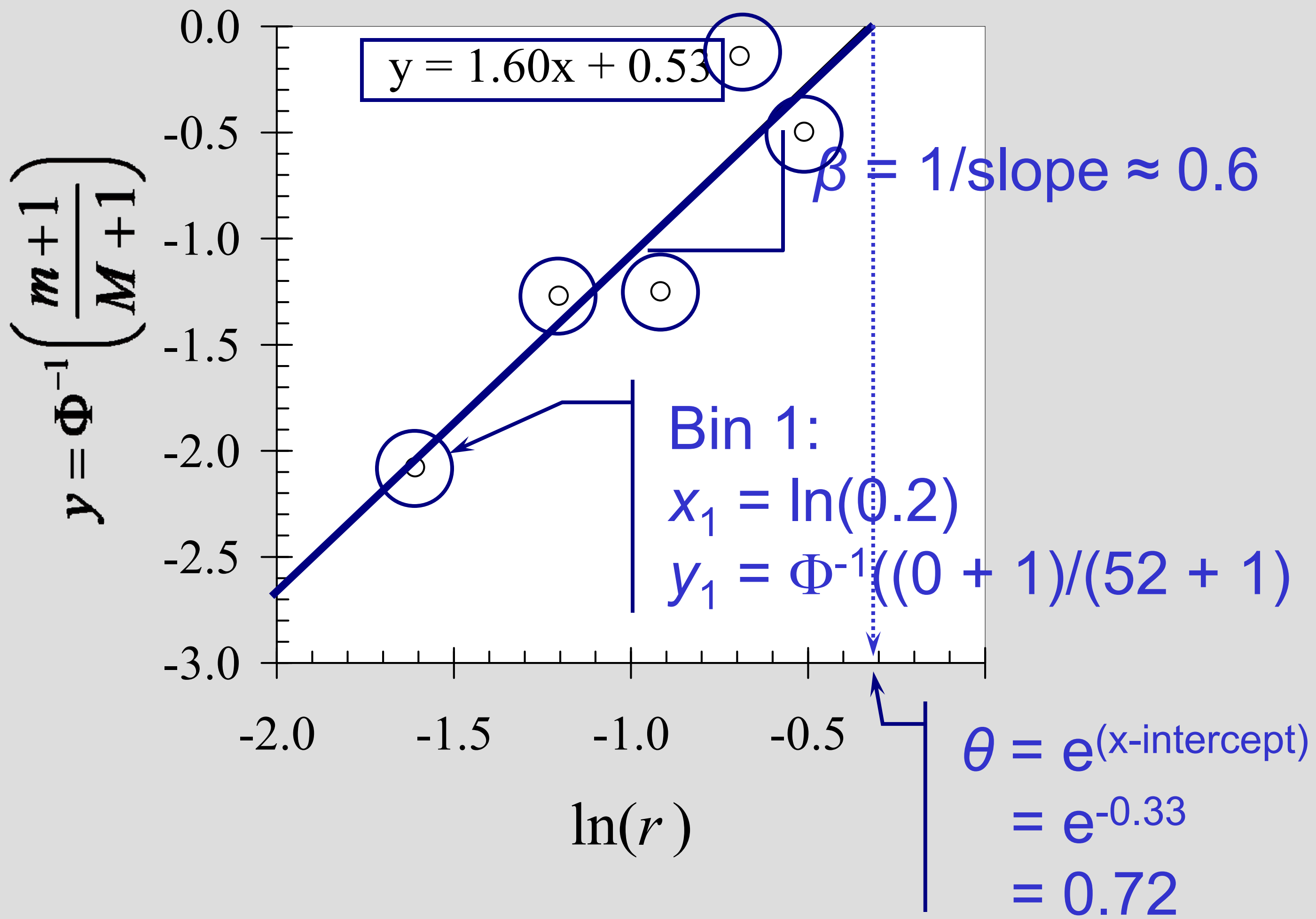
$\bar{x}$

$\bar{y}$

52 specimens (boxes in bin 1)

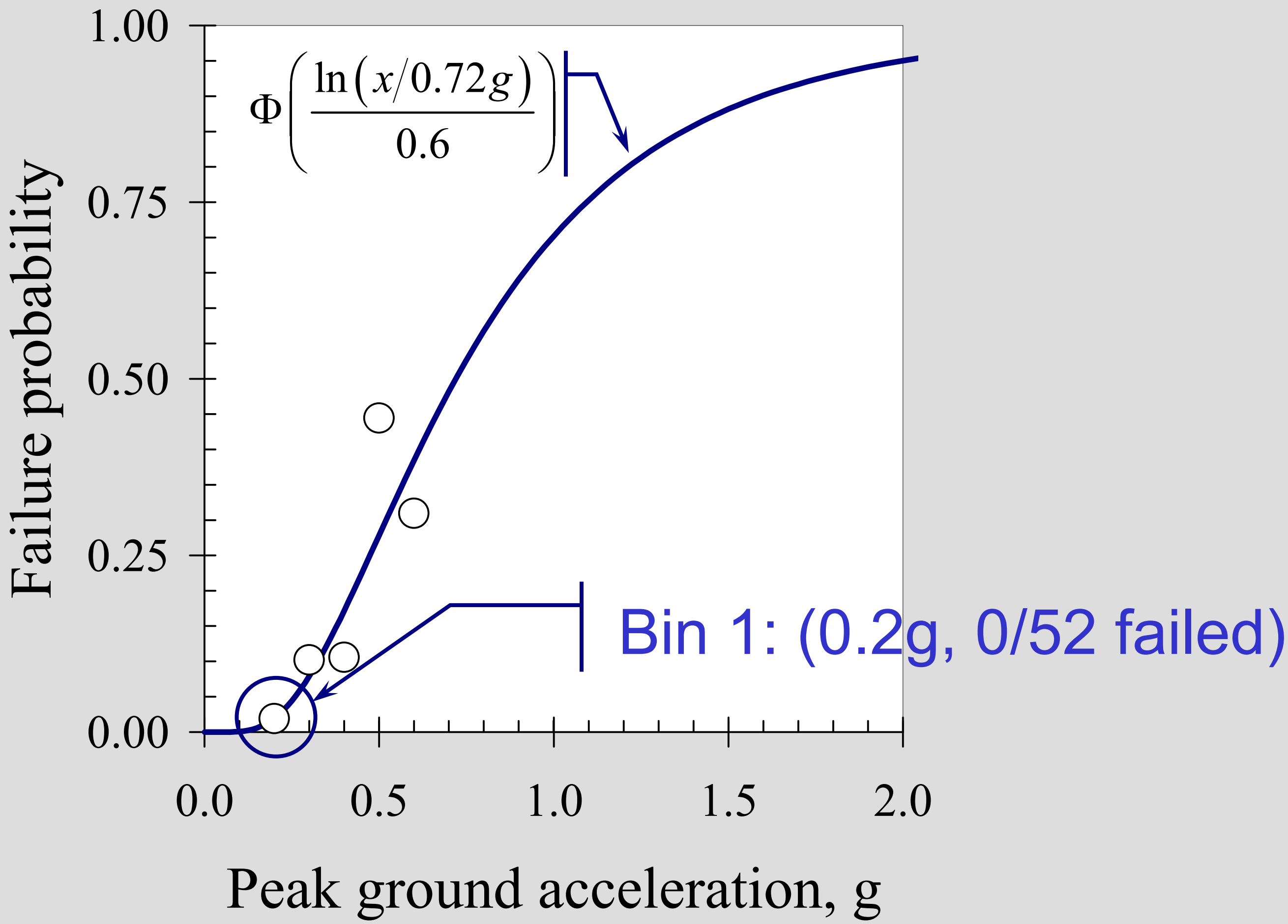
0 failed (black-filled boxes)

Method B illustration, continued



Method B illustration, concluded

Fragility function in DP-P_f space



Method B_2, weighted least squares

- N = number of bins
- i = index of bins, $i \in \{1, 2, \ldots N\}$
- Mi = number of specimens in bin i
- M = total number of specimens

$$M = \sum_i M_i$$

- m_i = number of failed specimens in bin i
- y_i = failure rate in bin i, i.e.,

$$y_i = \frac{m_i}{M_i}$$

Method B_2

- x_i = excitation in bin *i*
- w_i = subjective weight for bin *i*, i.e., the analyst's judgment of the degree to which the specimens or tests in bin *i* represent the general population of specimens. Default = 1.
- W = total weight

$$W = \sum_i w_i M_i$$

Method B_2

- Find θ and β_d to minimize ε, where

$$\varepsilon^2 = \frac{1}{W}\sum_{i=1}^{N} w_i M_i \left(y_i - \Phi\left(\frac{\ln\left(x_i/\theta\right)}{\beta_d} \right) \right)^2$$

$$\theta > 0$$

$$0.2 \le \beta_d \le 0.6$$

- Can use Excel solver. Then get *β*

$$\beta = \sqrt{\beta_d^2 + \beta_u^2}$$

Method C

No specimens failed, DP known

- Tabulate specimen data: (DP, any distress (Y/N))
- Create a bin of highest-DP specimens
  - DP ≥ 0.7 x (max DP of all specimens)
  - DP ≥ (min DP of any with distress)
- None failed but assign "subjective failure probs:"
  - f = 0.0 if no distress (no. of specimens = M_A)
  - f = 0.1 for some distress (no. of specimens = M_B)
  - f = 0.5 for distress suggesting imminent failure (M_C)
- Assign this bin average DP & failure probability:
  - x = (max DP) if no specimens had distress
  - = bin-median DP otherwise
  - y = preliminary failure prob. $(0.1M_B + 0.5M_C)/(M_A + M_B + M_C)$

Method C, no failures

r_i = excitation experienced by specimen i (i = 1, 2, ... M)

$r_{max} = \max_i\{r_i\}$

r_d = minimum excitation experienced by any specimen with distress

r_a = the smaller of r_d and $0.7 \cdot r_{max}$

M_A = no. specimens w/o distress and with $r_i \geq r_a$

M_B = no. specimens at any r_i with distress not suggestive of imminent failure

M_C = no. specimens at any r_i with distress suggestive of imminent failure

r_m = r_{max} if $M_B + M_C = 0$

= $0.5 \cdot (r_{max} + r_a)$ otherwise

Method C, no failures

S = subjective failure probability at r_m

$= (0.5M_C + 0.1M_B)/(M_A + M_B + M_C)$

$$\beta = 0.4$$

$$z = \Phi^{-1}\left(F_{dm}\left(r_m\right)\right)$$

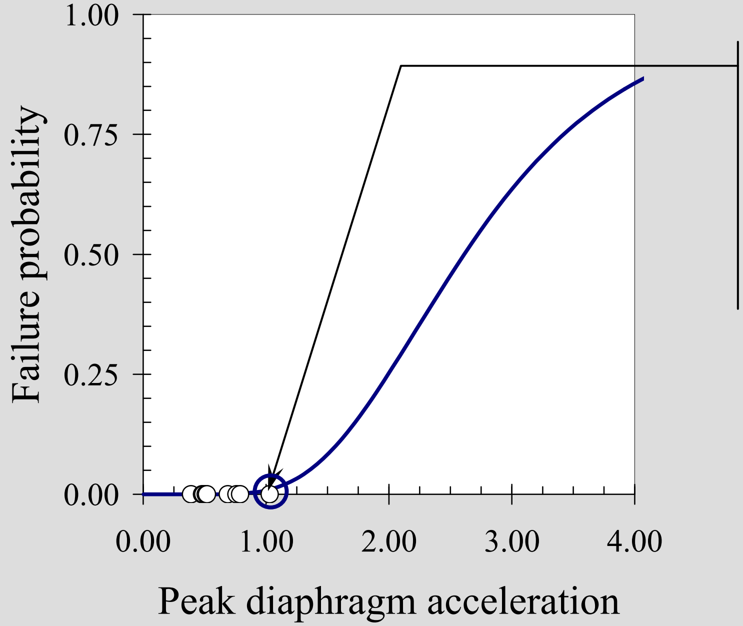
$$\theta = r_m \exp\left(-z\beta\right)$$

Conditions	$F_{dm}(r_m)$	Z	**exp(–zβ), β=0.4**
$M_A \geq 3$ and $S \leq 0.015$	0.01	-2.326	2.54
$M_A \geq 3$ and $S \geq 0.015$	S	$\Phi^{-1}(S)$	
$M_A < 3$ and $S \leq 0.075$	0.05	-1.645	1.93
$M_A < 3$ and $0.075 < S \leq 0.15$	0.10	-1.282	1.67
$M_A < 3$ and $0.15 < S \leq 0.3$	0.20	-0.842	1.40
$M_A < 3$ and $S > 0.3$	0.40	-0.253	1.11

Method C

No specimens failed, DP known

Case 1: No specimens with distress, several tested near max DP*



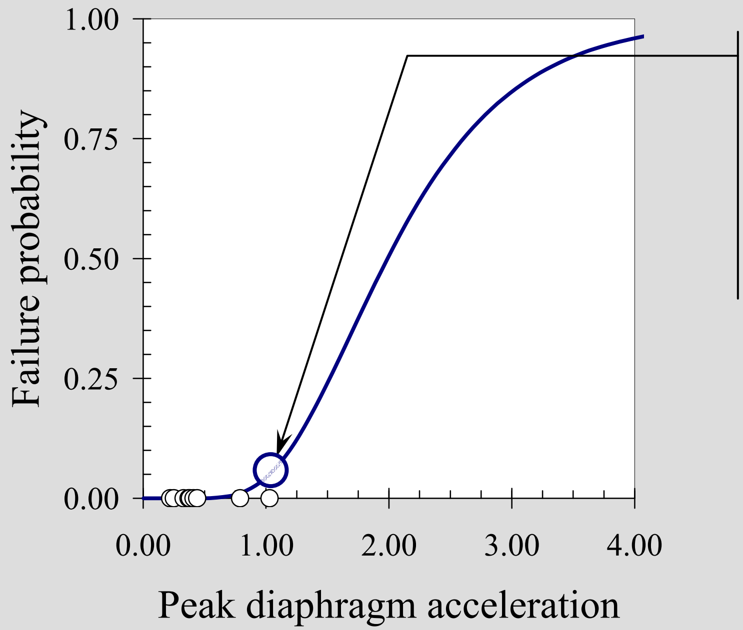
Assume 1% failure prob. at max DP and $\beta = 0.4$, so $x_m = 2.54 \cdot (\text{max DP})$

* 3+ specimens with DP $\geq 0.7 \cdot (\text{max DP})$

Method C

No specimens failed, DP known

Case 2: No specimens with distress, few tested near max DP*



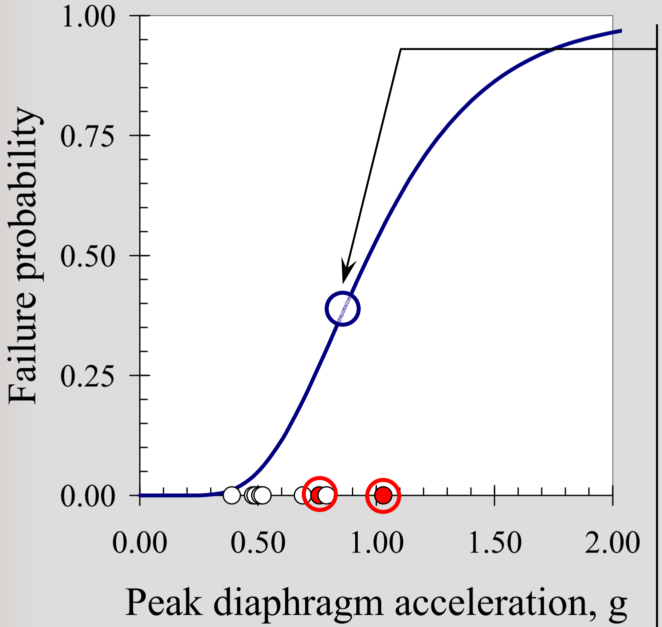
Assume **5%** failure prob. at max DP and $\beta = 0.4$, so $\theta = 1.93 \cdot (\text{max DP})$

* 1 or 2 with $DP \geq 0.7 \cdot (\text{max DP})$

Method C

No specimens failed, DP known

Case 3: Some specimens with distress



Treat specimens with	As if P_f =
severe distress	0.5
some distress	0.1
no distress	0.0

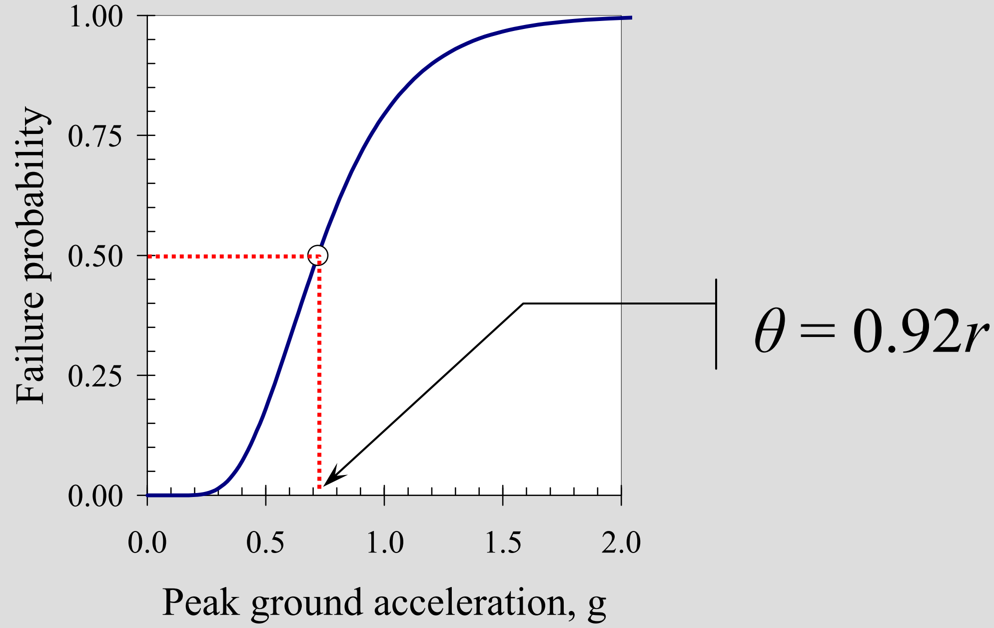
Calculate average *DP* and P_f of top-DP specimens, take β = 0.4, & fit curve through this point

Distress

Method D

Derived (analytical) capacity

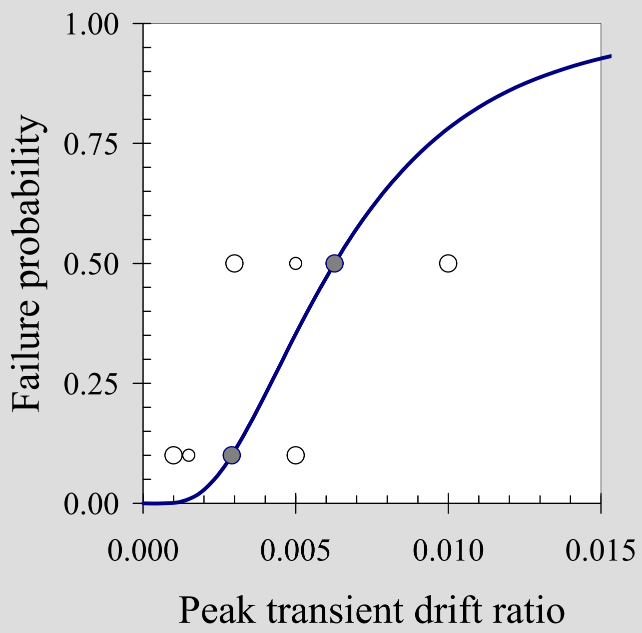
- No empirical failure data available
- Get (deterministic) capacity *r* from structural analysis
- Treat *r* as mean capacity, take $\beta = 0.4$, so $\theta = 0.92r$



Method E

Expert opinion

No data, too complex to analyze well



- Using a form, poll experts for
  - Median failure DP, θ
  - 10^{th} percentile DP, x_l
  - Level of expertise, w
- Calculate weighted average x_l, θ
- Fit curve through these

Method E

Expert opinion

Figure 1. Form for soliciting expert judgment on component fragility

Objective. This form solicits your judgment about the values of an engineering demand parameter (*EDP*) at which a particular damage state occurs to a particular building component. Judgment is needed because the component may contribute significantly to the future seismic performance a building, but relevant empirical and analytical data are currently impractical to acquire. Your judgment is solicited because you have relevant experience with the component of interest.

Definitions. Please provide judgment on the damageability of the following component. Images of a representative sample of the component and damage state may be attached. It is recognized that other *EDP*s may correlate better with damage, but please consider only the one specified here.

Component name: ______________________________

Component definition: ______________________________

____

Damage state name: ______________________________

Damage state definition: ______________________________

____

Relevant *EDP*: ______________________________

Definition of *EDP*: ______________________________

____

Method E

Expert opinion

Figure 1. Form for soliciting expert judgment on component fragility

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Component name: ______________________________

Component definition: ______________________________

____

Damage state name: ______________________________

Damage state definition: ______________________________

____

Relevant *EDP*: ______________________________

Definition of *EDP*: ______________________________

____

Uncertainty; no personal stake. Please provide judgment about this class of components, not a particular instance, and not one that you designed or otherwise have any stake in. There is probably no precise threshold level of *EDP* that causes damage, because of variability in design, construction, installation, etc., and even if there were, nobody might know it with certainty. To reflect uncertainty,

Method E
Expert opinion

Uncertainty; no personal stake. Please provide judgment about this class of components, not a particular instance, and not one that you designed or otherwise have any stake in. There is probably no precise threshold level of *EDP* that causes damage, because of variability in design, construction, installation, etc., and even if there were, nobody might know it with certainty. To reflect uncertainty, **please provide two values of *EDP* at which damage occurs: median and lower bound.**

Estimated median EDP: ______________ *Definition*: damage will occur at this level of *EDP* 5 times in 10. In a single case, you judge an equal chance that failure will occur at lower or higher *EDP*.

Estimated lower-bound EDP: ______________ *Definition*: damage will occur at this level of *EDP* 1 time in 10. In a single case, you judge a 10% chance that your estimate is too high. *Judge the lower bound carefully*. Make an initial guess, then imagine conditions that might make the actual failure *EDP* lower (errors in design, installation, deterioration, poor maintenance, interaction, etc.) and revise accordingly. Without careful thought, expert judgment of the lower bound tends to be too close to the median estimate, so think twice and do not be afraid of showing uncertainty.

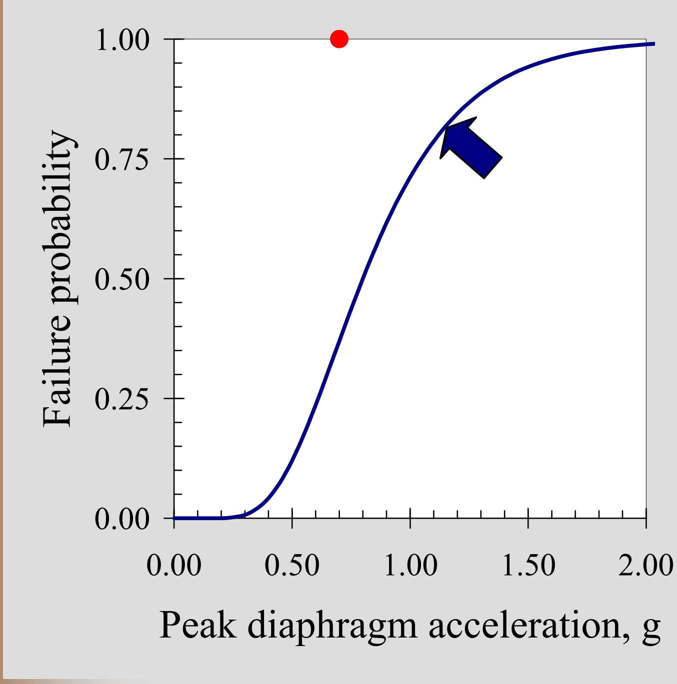
Your level of expertise (1-5): ______________ *Definition*: 1 means you have no experience or expertise with this component and damage state, 5 means you are very familiar or highly experienced.

Your name: ______________________________________ ***Date***: __________________

Method U

Fragility function exists, update with new data

Bayesian statistics provide a theoretical, consistent basis for updating a distribution based on new observations



- A prior fragility function exists
  - Its θ and β are treated as uncertain
  - Fragility function uses their mean vals
- New data arrive
- Calculate new distr. of θ and β
  - New fragility function uses mean θ, β
- Function moves toward new data

Goodness of fit

r^2 measures how well the regression accounts for the scatter in y

$$\sigma_{y|x}^2 = \frac{1}{M-1}\sum_{i=1}^{M}\left(y - F_{dm}(x)\right)^2$$

$$\sigma_y^2 = \frac{1}{M-1}\sum_{i=1}^{M} y^2$$

$$r^2 = 1 - \frac{\sigma_{y|x}^2}{\sigma_y^2}$$

Critical values of *r*

$r < r_\alpha$ → can't reject the "null hypothesis" that no trend exists between y and x

$f = DOF$

$f = n - 2$

n = no. data points

(subtract 2 for 2 parameter values)

Crow et al. (1960)

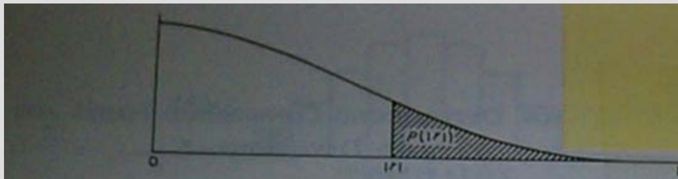


TABLE 7. CRITICAL ABSOLUTE VALUES OF CORRELATION COEFFICIENT r*

5% points and 1% points (in boldface) for equal-tails test of hypothesis $\rho = 0$.

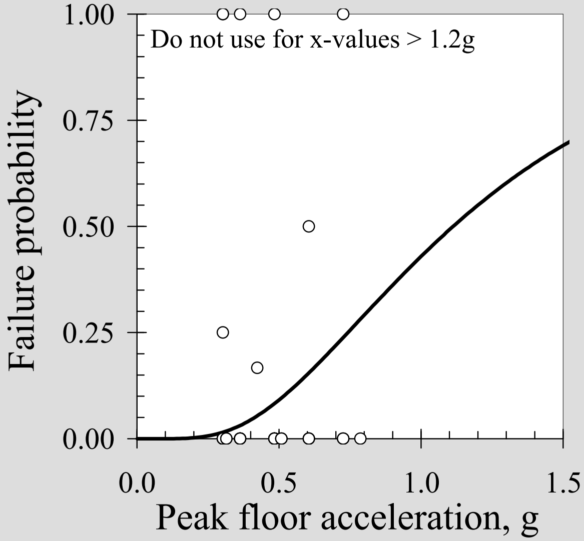
f	Total number of variables: 2	3	4	5
1	.997 **1.000**	.999 **1.000**	.999 **1.000**	.999 **1.000**
2	.950 **.990**	.975 **.995**	.983 **.997**	.987 **.998**
3	.878 **.959**	.930 **.976**	.950 **.983**	.961 **.987**
4	.811 **.917**	.881 **.949**	.912 **.962**	.930 **.970**
5	.754 **.874**	.836 **.917**	.874 **.937**	.898 **.949**
6	.707 **.834**	.795 **.886**	.839 **.911**	.867 **.927**
7	.666 **.798**	.758 **.855**	.807 **.885**	.838 **.904**
8	.632 **.765**	.726 **.827**	.777 **.860**	.811 **.882**
9	.602 **.735**	.697 **.800**	.750 **.836**	.786 **.861**
10	.576 **.708**	.671 **.776**	.726 **.814**	.763 **.840**
11	.553 **.684**	.648 **.753**	.703 **.793**	.741 **.821**

f	Total number of variables: 2	3	4	5
24	.388 **.496**	.470 **.565**	.523 **.609**	.562 **.642**
25	.381 **.487**	.462 **.555**	.514 **.600**	.553 **.633**
26	.374 **.478**	.454 **.546**	.506 **.590**	.545 **.624**
27	.367 **.470**	.446 **.538**	.498 **.582**	.536 **.615**
28	.361 **.463**	.439 **.530**	.490 **.573**	.529 **.606**
29	.355 **.456**	.432 **.522**	.482 **.565**	.521 **.598**
30	.349 **.449**	.426 **.514**	.476 **.558**	.514 **.591**
35	.325 **.418**	.397 **.481**	.445 **.523**	.482 **.556**
40	.304 **.393**	.373 **.454**	.419 **.494**	.455 **.526**
45	.288 **.372**	.353 **.430**	.397 **.470**	.432 **.501**
50	.273 **.354**	.336 **.410**	.379 **.449**	.412 **.479**

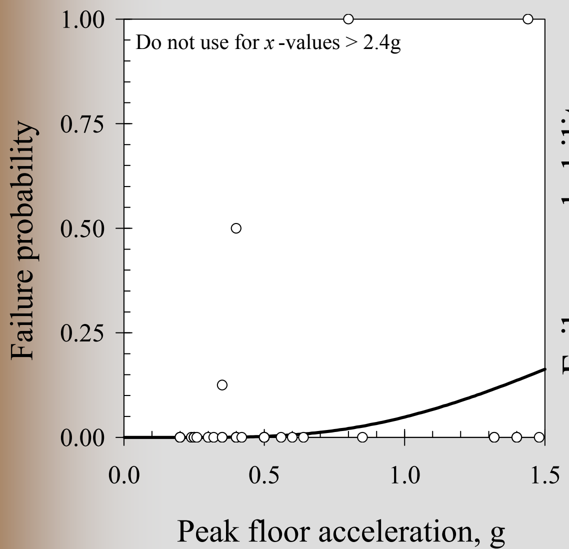
Goodness of fit

Some examples: battery racks

$$f = 81$$
$$r^2 = 0.59$$
$$r > r_{0.05}$$

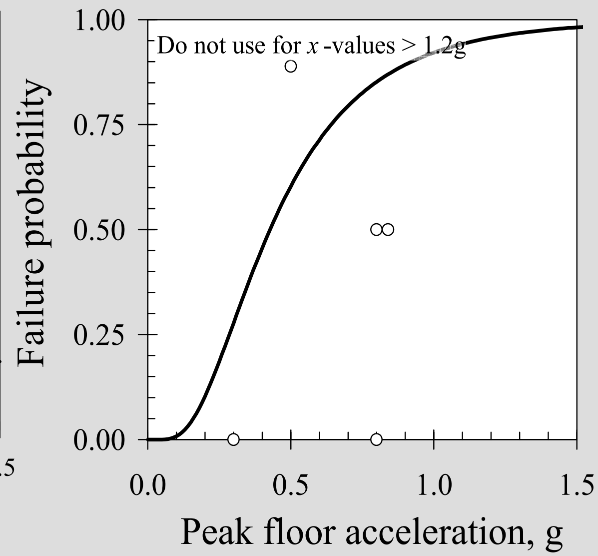


Goodness of fit, more examples



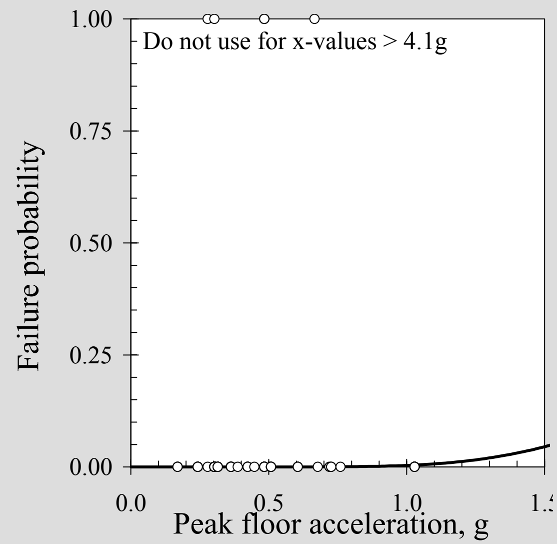
Battery chargers

$f = 87$
$r^2 = 0.0$
$r < r_{0.05}$



Chillers

$f = 25$
$r^2 = 0.84$
$r > r_{0.05}$



Control panels

$f = 131$
$r^2 = -0.1$
$r < r_{0.05}$

Implications of low r^2 values

- Intuition says there *should* be a trend
- Maybe low r^2 are from bad estimates of x?
- Signal-to-noise-ratio issue (low y at these x)?
- Maybe more data at higher x would give better fit?
- Maybe wrong demand parameter?
- Maybe component categories are by nature too diverse to justify estimating failure rate?

What are our choices?

- Instrument many buildings & wait. No progress from ANSS; no protocol exists.
- Lots of shake-table tests? Expensive & boring.
- Try different DP? Problem is lack of x-data.
- Rely on expert opinion or on unpublished data? Like accepting an alternative because it is harder to check.
- HCLPF approach? Rare, serious deficiency could bias results.
- Give up?
- Accept that fragility functions may meet some requirements but not all.
  - Monotonically increase with x
  - Have a convenient and traditional form
  - Generally pass through the cloud of data
  - Sometimes no better than does a flat line through the average

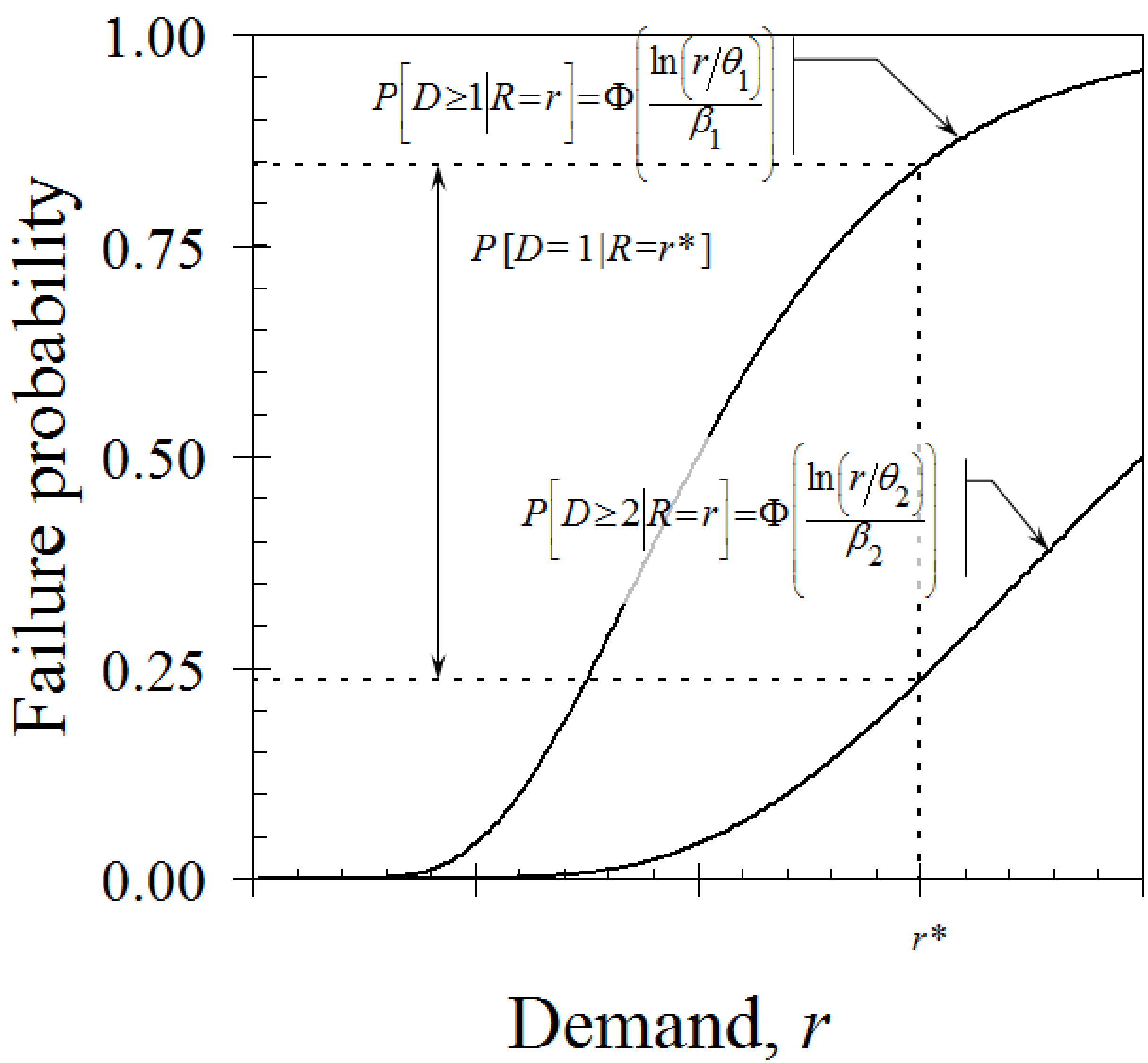
Multiple damage states

- Sequential: must pass through DS_i to reach DS_{i+1}, or repair of DS_{i+1} repairs DS_i

$$P\left[D=d \middle| R=r\right] = 1-\Phi\left(\frac{\ln\left(r/\theta_1\right)}{\beta_1}\right) \qquad d=0$$

$$= \Phi\left(\frac{\ln\left(r/\theta_i\right)}{\beta_i}\right) - \Phi\left(\frac{\ln\left(r/\theta_{i+1}\right)}{\beta_{i+1}}\right) \qquad 0<d<N_D$$

$$= \Phi\left(\frac{\ln\left(r/\theta_{N_D}\right)}{\beta_{N_D}}\right) \qquad d=N_D$$


1.00
0.75
0.50
0.25
0.00
Failure probability
$P\left[D \geq 1 \middle| R = r\right] = \Phi\left(\frac{\ln\left(r/\theta_1\right)}{\beta_1}\right)$
$P\left[D = 1 \middle| R = r^*\right]$
$P\left[D \geq 2 \middle| R = r\right] = \Phi\left(\frac{\ln\left(r/\theta_2\right)}{\beta_2}\right)$
r^*
Demand, r

Multiple damage states

- MECE: only 1 DS occurs if component fails

$$P\left[D=d\,\middle|\,R=r\right]=\Phi\left(\frac{\ln\left(r/\theta_1\right)}{\beta_1}\right)\cdot P\left[D=d\,\middle|\,d>0\right]$$

$$D=i\rightarrow D\neq j,\quad i\neq j$$

$$\sum_{d=1}^{N_D}P\left[D=d\,\middle|\,d>0\right]=1$$

- Simultaneous: failure implies 1 or more DS

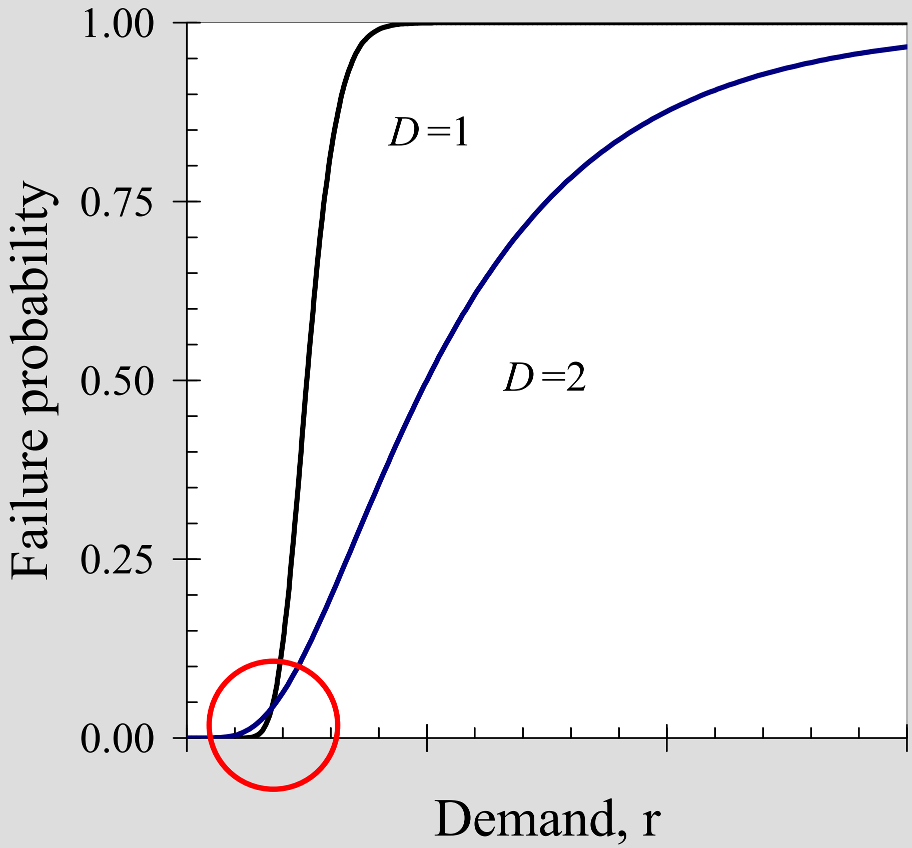
$$P\left[D=d\,\middle|\,R=r\right]=\Phi\left(\frac{\ln\left(r/\theta_1\right)}{\beta_1}\right)\cdot P\left[D=d\,\middle|\,d>0\right]$$

$$\left(D=i\right)\cap\left(D=j\right)\neq\phi\qquad i\neq j$$

$$\sum_{d=1}^{N_D}P\left[D=d\,\middle|\,d>0\right]>1$$

Fragility functions that cross

- Happens if $\beta_i \neq \beta_j$, $i \neq j$
- But P[D=1] ≥ 0 ∀ *r*
- Ad-hoc solutions:
  - Force P[D=1] ≥ 0
  - Require $\beta_i = \beta_j$



Method B doesn't converge

- It is common to see failures at low *r*, none at high *r*, and B_2 won't converge
- Can use method C as a backup, treating failures as "damage indicative of imminent failure"

Other distributions

- Normal $$P\left[D \geq d \middle| R = r\right] = \Phi\left(\frac{r/\theta}{\beta}\right)$$
- Gumbel type 1 (minimum) $$P\left[D \geq d \middle| R = r\right] = 1 - \exp\left(-\exp\left(r\right)\right)$$
- Gumbel type 2 $$P\left[D \geq d \middle| R = r\right] = 1 - \exp\left(-b \cdot r^{-a}\right)$$
- Gamma…
- Other distributions: higher r^2 is better*

(*for the same number of parameters)

Qualitative measures of quality

Data quality: number of data points, coverage over the range of damage states, constraints and means of observing x and y; number of independent sources;

Data relevance: how well the data matches or envelopes the conditions encountered in real buildings; diversity of exposed types; bias in the selection of observations;

Documentation quality: how well the author has documented the data, the analysis, and the results;

Rationality: how well the behavior can be explained or rationalized by intuition, calculation, or principles of engineering mechanics.

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Questions?

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